NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN INVENTORY MODEL
FOR THE
PAKISTAN NAVAL STORE DEPOT

by

Ahmad Hayat

June 1985

Thesis Advisor: Alan W. McMasters
Co-Advisor: J. W. Creighton

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) An Inventory Model for the Pakistan Naval Store Depot		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; June 1985
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ahmad Hayat		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE June 1985
		13. NUMBER OF PAGES 84
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inventory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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S N 0102- LF- 014- 6601

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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An Inventory Model for the Pakistan Naval Store Depot

by

Ahmad Hayat
Commander, Pakistan Navy
B.S., Pakistan Naval Academy, 1969.
M.B.A., Golden Gate University, San Francisco, Ca., 1985.

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MANAGEMENT

from the

NAVAL POSTGRADUATE SCHOOL
June 1985

ABSTRACT

The complications involved in maintaining at least five diverse inventories from five different countries is a gigantic task for the Pakistan Navy. This task can be considerably eased by the introduction of a forward looking and a scientific inventory management approach to systemize the Pakistan Navy's inventory management requirements. This thesis offers a simplified version of the consumable inventory model used by the United States Navy inventory system with modifications to make it appropriate for the Naval Stores Depot of the Pakistan Navy. Emphasis has been placed on minimizing the sum of the average annual costs of ordering, carrying, and shortages. The procedures for determining the optimal order size and reorder point for an item are provided along with the steps for implementing the model.

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I. INTRODUCTION

A. MOTIVATION

"If inventory management is everybody's business, why does it cause so many problems?" [Ref. 1]

Clearly, inventory management at present is not everybody's business.

A need exists to improve the present concepts and the present methodologies that are in use in the Pakistan Navy for managing inventories. The Pakistan Navy, like most Navies of the impoverished third world nations is supported by an anemic national economy. Financial constraint is a singular factor which plagues all facets of the country. Therefore, the Pakistan Navy has to be very careful in spending each Rupee or Dollar (our foreign transactions are done in Dollars). In addition, the constraint of foreign exchange has resulted in acquisition of inventories of spare and repair parts from diverse countries. This also carries with it the disadvantage of long and--at times--unpredictable lead times for the items requisitioned. In turn, this necessitates building up a high level of inventory. The introduction of scientific inventory management concepts should help the Pakistan Navy better commit its resources.

B. EVOLUTION OF INVENTORY MODELS

Inventory is usually the major asset and generally one of the most maneuverable, other than cash itself. A basic conflict, however, exists in the functional segments of most organizations. This conflict is in their view of inventory needs in the light of their own objectives. However, most of these conflicts evaporate when examined in the light of sound economic judgements, and most inventory policies can be established through cost optimization principles. However, this in itself leads to another relationship problem. The increased use of mathematical techniques in an area once thought to be governed by experience and common sense is still being met with resistance--especially by the old timers, whose strength is their experience and not their flexibility and rationale view of innovations.

Such a situation calls for increased sensitivity on the part of the top management, while trying to project and implement these techniques. It is important that the knowledge base of all concerned be developed before an inventory system, of what ever level of sophistication or complexity it may be, can be introduced in the system. It is one thing to introduce a system and entirely another to obtain cooperative responses from the various rungs of the hierarchy. In introducing any system, a great potential exists for alienating people who may not be motivated towards that system. The result is that the advantages of the new system may be offset by the hostility of its implementors.

During the last three decades there has been a rapid growth of interest in what is often referred to as scientific inventory control [Ref. 2] Scientific inventory control is generally understood to be the use of mathematical models to obtain rules for operating inventory systems. The subject has attracted such wide interest that today every serious student in the management science or industrial engineering area is expected to have some knowledge of

inventory models. Originally, the development of inventory models had practical application as its immediate objective. To a large extent this is still true, but as the subject becomes more mature, better developed, and more throughly explored, an increasing number of individuals are working with inventory models because these present interesting theoretical problems in mathematics. Such individuals are making a major contribution to the future of inventory management by making available more and more versatile applications of inventory related models. Thus today work is being done with inventory models at many different levels, ranging from concerns only for practical problems to concerns only for the mathematical properties of the model, projecting into the future needs.

Although inventory problems are as old as history itself, it has only been since the turn of the century that any attempts have been made to employ analytical techniques in studying these problems. The initial impetus for the use of mathematical methods in the inventory analysis seems to have been supplied by the simultaneous growth of the manufacturing industries and the various branches of engineering--especially industrial engineering. The real need for analysis was first recognized in industries that had a combination of production scheduling problems and inventory problems, i.e., in situations in which items were produced in lots--the cost of set up being fairly high--and then stored at a factory warehouse.

The earliest derivation of what is often called the simple lot size formula was provided by Ford Harris of the Westinghouse Corporation in 1915. [Ref. 3] This formula has been developed, apparently independtly, by many individuals since then. It is often referred to as the Wilson formula since it was also derived by H. R. Wilson as an integral part of an inventory control scheme which he sold to many organizations. The first full length book to deal with inventory problems was that of F. E. Raymond, written

while he was at M.I.T. It contains no theory or derivations, and only attempts to explain how various extensions of the simple lot size model can be used in practice.

It was not until after World War II, when the management sciences and operations research fields emerged, that detailed attention was focused on the stochastic nature of inventory problems. Prior to that the problems had been treated as if they were deterministic, except for a few isolated cases, such as the work of Wilson, where some attempts were made to include probabilistic considerations.

The original requirement for using analytical techniques to solve inventory problems arose in industry where engineers were seeking solutions to practical problems. It is interesting to observe that economists were not the first to take an active interest in inventory problems even though inventories play a crucial role in the study of dynamic economic models. However, interests of both the mathematicians as well as the economists were eventually kindled in inventory models. This interest primarily evolved from the very real "need to know" of the engineers. Now both these schools actively participate in making contributions to the inventory field.

With time the development and applications of inventory models have become wide spread. Computers have played a significant role in the applications of inventory models--ranging from an extensive and at times very progressive and innovative use by the military services to its pioneers; that is, the manufacturing levels and retail industries.

C. INVENTORY CONTROL

Inventory control is the science-based art of controlling the amount of stock held, in various forms, within a business to meet economically the demands placed upon that business. [Ref. 4]

Such a business can vary from that of a Naval Stores Depot, charged with the responsibility of providing material to units within a certain radius, to the international company operating throughout the world and subjected to domestic and foreign demands, and to the small specialist firm supplying a larger organization. Inventory control has important applications in nearly all facets of business as well as non-profit organizations. The generics of the control system remain the same wherever they are applied. What changes is the magnitude of responsibilities and the range of variables and, therefore, the range of the inherent complexities.

Stocks held by a business can take many forms. The well known forms are either finished product stocks--that is the items waiting for sale or issue on demand--or the raw material stocks needed to support manufacturing. However, in between these two types are all the in-process stocks which occur naturally as part of the production process.

Top commands in most military services, like the top management in most multinational corporations, are becoming increasingly aware that the overall efficiency of their service or their company's operation is directly related to inventory situations existing within the organization. Thus, there has been an increasing requirement for a knowledge of the mathematical inventory theory which can be used to analyze and control stocks. The United States military services have been managing their inventory since the mid sixties with the aid of such mathematical models.

D. PURPOSE AND SCOPE

In the Pakistan Navy, inspite of the reasonably good inventory control procedures, there is potential to apply scientific inventory control models. The incorporation of such an inventory model should help provide a better range and depth of spares for the

use of the fleet. This in turn should reduce down time of the concerned equipment, thus making possible a higher ratio of operational utilization of the parent equipment. The purpose of this thesis is to suggest an inventory model for managing consumable items. A second purpose is to present it in such a way that inventory managers can understand its logic and facilitate its implementation. It is in keeping with this aim that both the discussion and the approach has been kept as simple as the elements of inventory theory will allow. Effort has been made to keep away from complex day-to-day inventory management problems and address only the foundations of this otherwise gigantic pyramid.

E. PREVIEW

In Chapter II the author introduces the inventory management concepts. In talking about demand based inventories and its operating doctrine, the total annual cost of inventory is discussed. A simple example on the subject illustrates the ingredients and the concepts of cost tradeoffs. This chapter closes with a discussion of non-demand based inventories.

In Chapter III the simple EOQ model is developed. This leads to the total annual cost formula and other formulas to obtain the order size and the reorder point. A basic assumption of the model is of certainty of demand. This is followed by a discussion of an uncertainty of demand model and how the various costs differ from the EOQ model. Elements of safety stock and backorder are introduced. Finally, formulas for computing the optimal order size and reorder point are discussed.

Chapter IV deals with forecasting and demand distributions. The moving average model for estimating the mean value of quarterly demand is discussed, followed by a discussion of the exponentially weighted average model. A model for forecasting

variation in quarterly demand is discussed next. Finally the forecasting of lead time is considered. The two forms of probability distribution, the normal standard distribution and the Poisson distribution, which are most commonly used in the U.S.Navy to describe the distribution of demand during lead time are discussed in the closing section of this chapter.

Chapter V deals with providing initial backup supply support to complex new weapon systems or equipment. Demand forecasting models are discussed first. A model called the Cost Difference Formula, is then considered to determine the range of items to buy. Finally, a model for computing the buy quantity for those items to be bought is presented and steps for computing the procurement budget are described.

Chapter VI presents a summary of the thesis. It also presents conclusions and recommendations for implementing the models in the thesis.

II. INVENTORY MANAGEMENT CONCEPTS

A. INTRODUCTION

In an ideal world where the demand upon a business is well known in advance and where suppliers keep to their due dates, there would be little need for a business to hold any form of inventory other than a limited amount of in-process stocks caused as a by-product of the manufacturing process. Such an inventory problem would be completely deterministic because all the problem parameters would be exactly defined.

In practice however, demand is not known in advance and suppliers' delivery times are not known precisely. In this imperfect but practical situation, stocks can act as a buffer between the vagaries of supply and demand.

Of the many reasons for carrying an inventory one is [Ref. 3]:

> "Inventories make it possible to perform missions or tasks independently without relying on production facilities. Manufacturers use inventory to separate or decouple one production capability from another, while in the Navy's case, inventories allow fleet operations to be carried out in remote locations for long periods of time."

In the Pakistan Navy there are basically two categories of items, Consumable and Repairable. Repairable items are permanent in nature and can either be repaired at the organization level or are surveyed (sent) to the base facility for repairs. The

surveying organization can, on the authority of its survey note, demand another item from the depot as a replacement.

The Pakistan Naval Stores Depot is the centralized issuing agency for the entire Navy. All units irrespective of their location submit their demands (requisitions) to the Naval Stores depot. Similarly, all items being surveyed are sent to the Naval Stores Depot.

Consumable items, as the name indicates, are consumed in use or cannot be economically repaired when they fail to function. This classification of items need not, therefore, be surveyed to the depot by the ship when requesting a replacement item.

The basic assumption of all inventory models is that a demand estimate is available for each item managed. This estimate is typically obtained from historical demand data. Based upon this information the items may then be classified as demand based or non-demand based items. This chapter will first consider the theory associated with demand based items since these dominate the inventories of the Pakistan Navy. The procedures for handling non-demand based items will be discussed at the end of the chapter.

B. DEMAND BASED INVENTORIES

1. *Operating Doctrine*

The primary questions in the demand based inventory are, "how much to order?" and, "when to order?". The answer to these questions constitute what we will call "the operating doctrine". Under ideal circumstances, with no constraints of funds and storage space these two questions could be easily answered by either placing one very large order, (enough so as not to run out of

stock in the foreseeable future) or, if the lead time is minimal, order as frequently as demand occurs.

Unfortunately, in the real world of scarce resources, the financial constraints are pressing and do not permit the luxury of an arbitrary ordering pattern. The inventory manager, therefore, must consider the cost and availability of storage space, the cost to place an order, the cost of material purchased and the inventory obsolescence rates. In addition, both the Navy as well as the private sector want to keep their customers satisfied. In the private sector satisfied customers mean increased sales; in the Navy, satisfied customers mean enhanced combat readiness of this National defence force. Thus, a lack of service incurs some penalty cost. Selecting an operating doctrine which balances these costs tends to minimize the average annual total cost of managing inventories.

C. TOTAL ANNUAL COST

The total average annual costs equation can generally be described as:

Total Annual Costs = Annual Cost of Buying the Parts (eqn 2.1)
+ Annual Admin Order Costs
+ Annual Holding Costs
+ Annual Shortage Costs

Here the annual cost of buying parts is considered a fixed cost because it depends only on the annual demand and not on the operating doctrine. However, the inventory manager has control over the order cost, the holding cost and the shortage cost through his decisions of what to buy, how much to buy and when to buy.

1. *Administrative Order Costs*

This consists of many different elements; e.g., the salaries of personnel involved in inventory control and contracting, supplies and data processing costs used to determine and process buys, and the costs of receiving. Here the cost of receiving involves the paper work part of the transaction and does not include actual materials handling.

It may also include the cost associated with the following operations [Ref. 6]:

1) Review of the level of stock in inventory.
2) Preparing and processing an order requisition or purchase request.
3) Selection of a supplier.
4) Preparation and processing the purchase order.
5) Preparing and processing receiving reports.
6) Checking and inspecting stock.
7) Preparing and processing payment.

2. *Setup Costs*

The setup cost replaces the ordering cost in manufacturing concerns where, for each new production run, setup time is required. Setup costs are those expenses that are incurred each time a company lays out a production line to produce a different item for inventory. Such expenses include the opportunity cost associated with the delay in setting up the line and the associated personnel costs. In some instances, such expenses are referred to as change-over costs.

3. *Carrying Costs*

This is also called the holding cost. Elements of the carrying or holding costs are investment or opportunity cost of capital, the cost of losses due to obsolescence and other losses such as pilferage, material handling and other warehousing costs. Holding or carrying costs are usually expressed as a percentage of the average annual on-hand inventory value.

The magnitude of such costs varies considerably. For example, raw materials can often be dumped out of rail cars and stored outside, whereas finished goods require safer handling and more sophisticated storage facilities. In some instances warehousing cost is delineated separately from carrying cost. In the private sector carrying costs usually include insurance and taxes. However, these latter costs are not usually applicable to Navy inventories. In exceptional circumstances some very valueable inventory may be insured but taxes are never of concern.

The depreciation and obsolescence elements are of particular importance to the Navy. While goods are held in storage, there is a chance that they might depreciate in value, become obsolete, or even experience damage when transported to or handled in the warehouse. Changes in styles and technology render goods in inventory less desireable and, therefore, less valueable and are a major concern of military organizations.

The final major component of carrying cost is interest or opportunity cost. In other words, what it costs us to have capital tied up in inventory.

Since interest or opportunity cost on inventory investment usually account for approximately half of the total holding cost in the military, and are usually expressed as a percentage rate of the value of inventory, the cost estimate representing all elements of holding cost is usually expressed the same way.

4. *Shortage or Backorder Costs*

These costs are rather difficult to ascertain. The cost means a loss of service to a Navy customer. In case of a business enterprise it would typically mean a monetary loss for the firm. In the case of the Navy, or any other service-oriented agency not working on generation of revenues, this cost is difficult to ascertain because it is difficult to say what it actually costs the Navy if the radar of one of its ship is down for a couple of days. Or what is the monetary effect if a ship has not been able to go out to sea for a few days due to lack of spare parts. Here, the value of not being operational depends on what time frame is relevant. If it is a wartime scenario, the "costs" associated with a lack of a spare part would be far greater than in normal peace time. Thus, the shortage cost should somehow be associated with mission criticality.

D. COST TRADE-OFFS

1. *Ordering and Carrying Costs*

The definitions and the preceding discussions established that the goal of a good inventory manager, be it in the Navy or in the business world, should be to minimize the average annual variable costs. The manager should, therefore, be aware of when to buy and in what quantity to buy so as to keep the three variable costs (costs of ordering, carrying, and shortage) to a minimum. To accomplish this a manager has to make trade-offs between the three variable costs. He has to weigh each against the others and establish a combination that will reduce the overall costs.

As must be evident by now, the ordering costs and the carrying costs act in opposite ways. The larger the size of

orders, the less frequently will they be needed to be made, and the smaller will be the average annual order cost associated with them. On the other hand, the larger the quantity ordered the larger would be the average on-hand inventory and hence the carrying costs associated with it. So the optimal order quantity should result from an efficient trade-off between these two costs. The following example should help in illustrating this cost trade-off.

Assumptions:
The order cost is Rs.(Rupees) 200 for each order placed.
One week's demand is 100 units;
Value per unit is Rs 100;
Percentage carrying cost is 25%;
Order cost is Rs 200.

In the following tables and figures first the order cost, then the carrying cost and finally a combination of the two costs are shown. Table I shows the relationship between the number of orders per year and the order size or the Q value associated with each. The order size has been computed under the assumption that each week's demand is 100 units and that all demands must be met. If we place an order every week of the year then we would place 52 orders a year, there being 52 weeks in a year. On the other hand, if we decided to get our annual need of the inventory in one order, then we would place one order every 52 weeks. This order size is determined by dividing the annual demand of 5200 by the number of orders per year.

Table II represents the order costs for the various number of orders placed per year. Because the number of orders is directly related to the order size we can control the frequency of orders by the quantity we order each time we place an order. Therefore, we have chosen to show order size rather than the order frequency in Table II. The reader can refer to Table I to get the latter.

Table II and Figure 2.1 show that the larger the order the less is the annual order cost. Since it costs the system Rs 200 to place an order, it will be most cost effective to place just one order each year.

Table III and Figure 2.2 show the relationship between the size of an order and the annual carrying cost. The average on-hand inventory is half the value of the order size quantity because the on-hand inventory varies between a maximum of Q and a minimum of zero. The annual carrying cost is computed as the product of the percentage carrying cost, the value of one unit, and the average on-hand inventory.

Figure 2.2 shows that the carrying costs are a linear function of order size. In other words it rises directly as the level of average inventory rises. Therefore, the smaller the order size, the lower the average on-hand inventory and hence the lower the carrying costs.

Table IV and Figure 2.3 show the sums of the annual order and carrying costs for each order size and illustrates the trade-off between these two costs. The table shows that the minimum total annual costs are not at any one of the two extremes, carrying very high inventory to keep the number of orders small or keeping a very low on-hand inventory by placing many orders a year.

TABLE I

Order Frequency and Order Size

No. of orders per year	Order size
52	100
36	144
26	200
18	288
15	347
13	400
12	433
8	650
5	1040
4	1300
3	1733
2	2600
1	5200

TABLE II

Order Size and Order Costs

Order size	Total Annual Order Costs
100	Rs.10,400
144	7,200
200	5,200
288	3,606
347	3,000
400	2,600
433	2,400
650	1,600
1,040	1,000
1,300	800
1,733	600
2,600	400
5,200	200

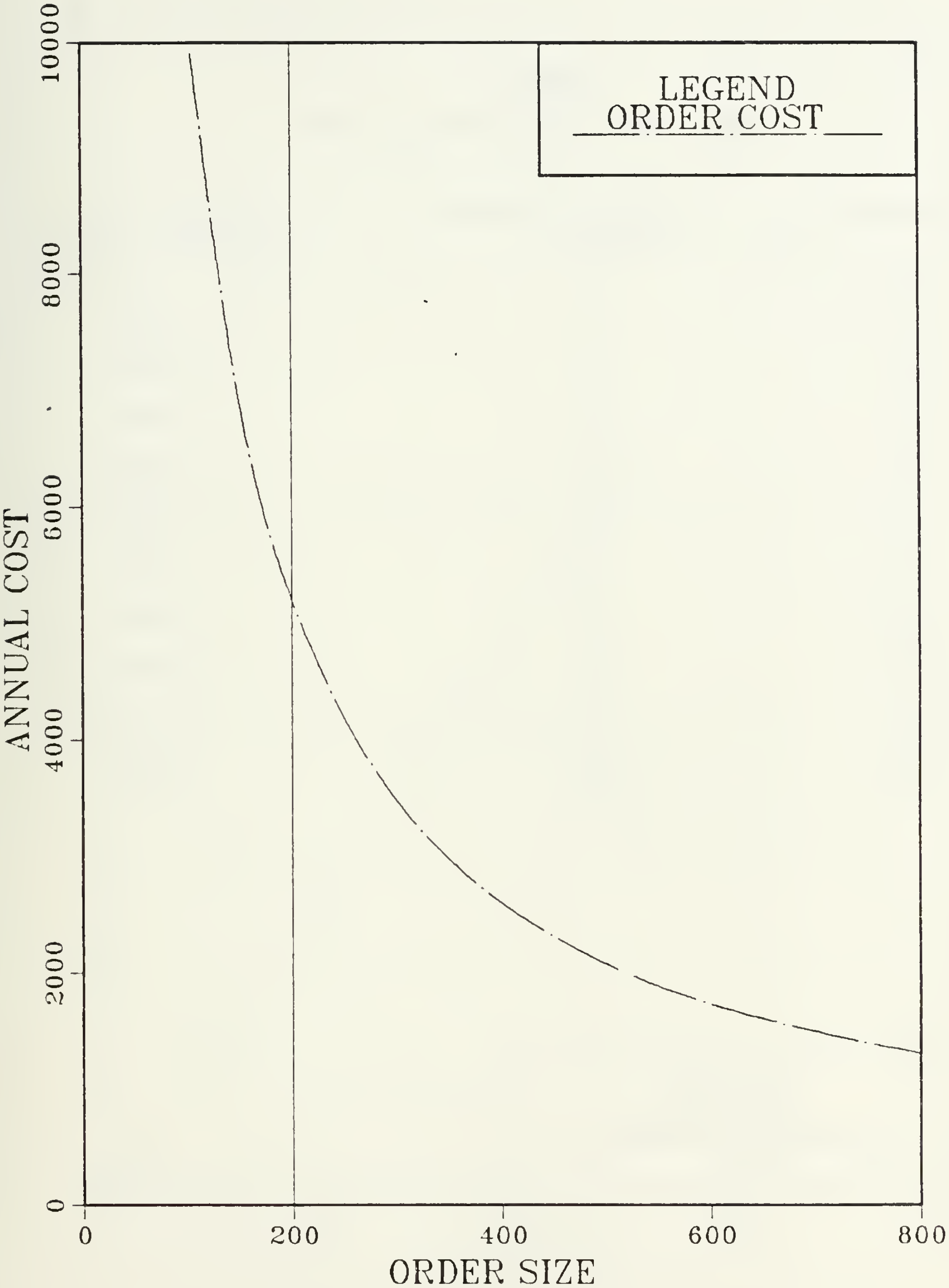

Figure 2.1 Total Annual Order Costs versus Order Size

TABLE III

Order Size and Carrying Costs

Order Size	Average on-Hand Inventory	Total Annual Carrying Costs
100	50	Rs 1,250
144	72	1,805
200	100	2,500
288	144	3,605
346	173	4,334
400	200	5,000
433	217	5,415
650	325	8,125
1,040	520	13,000
1,300	650	16,250
1,733	867	21,665
2,600	1,300	32,500
5,200	2,600	65,000

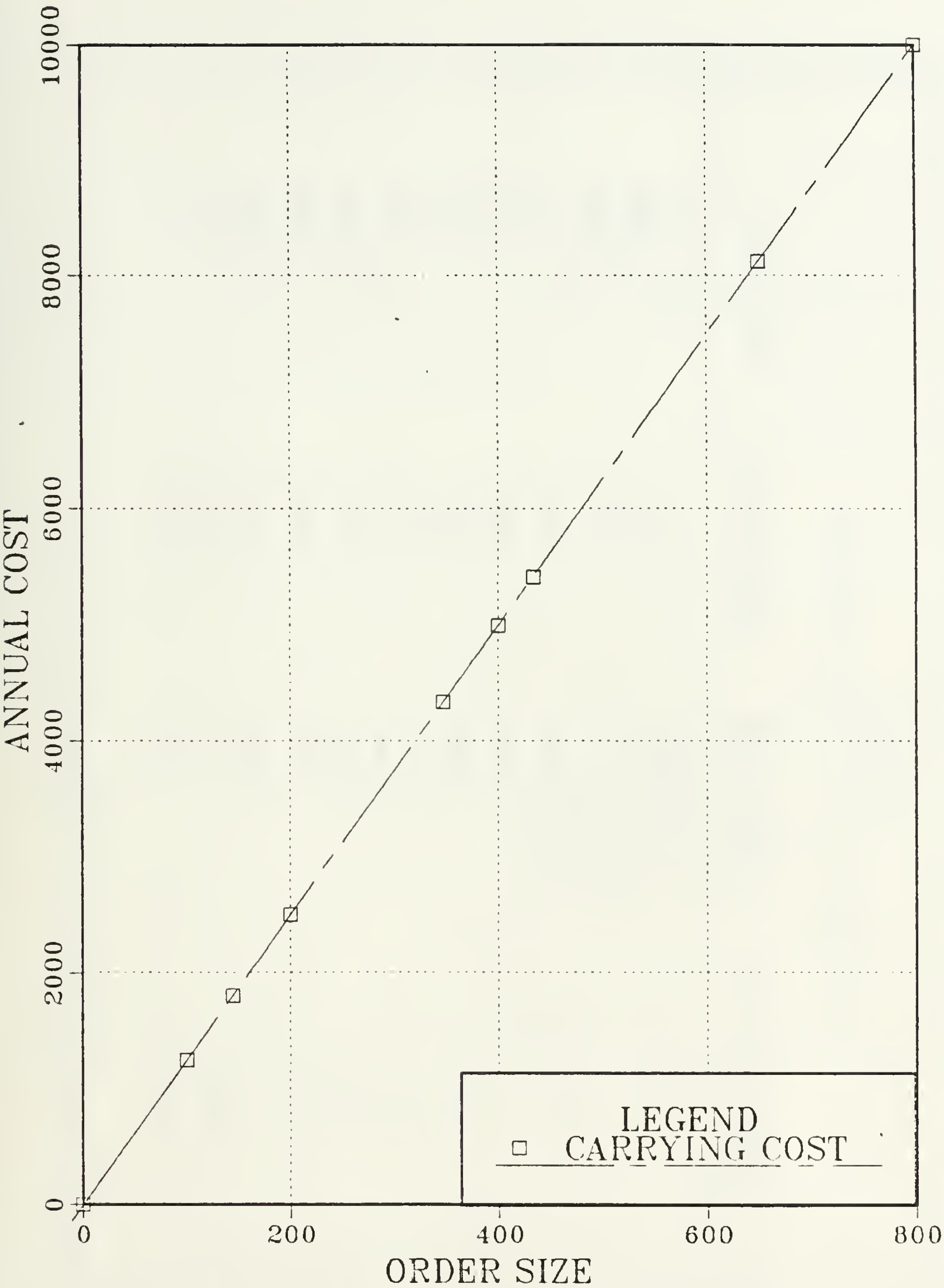

Figure 2.2 Total Annual Carrying Cost versus Order Size

TABLE IV

Components of the Total Annual Costs

Order Size	Average on-hand inventory	Total Annual Order Costs	Total Annual Carrying Costs	Total Costs
100	50	Rs 10,400	Rs 1,250	Rs 11,650
144	72	7,200	1,805	9,005
200	100	5,200	2,500	7,700
288	144	3,606	3,605	7,211
347	173	3,000	4,334	7,334
400	200	2,600	5,000	7,600
433	217	2,400	5,415	7,815
650	325	1,600	8,125	9,725
1,040	520	1,000	13,000	23,000
1,300	650	800	16,250	17,050
1,733	867	600	21,665	22,265
2,600	1,300	400	32,500	32,900
5,200	2,600	200	65,000	65,200

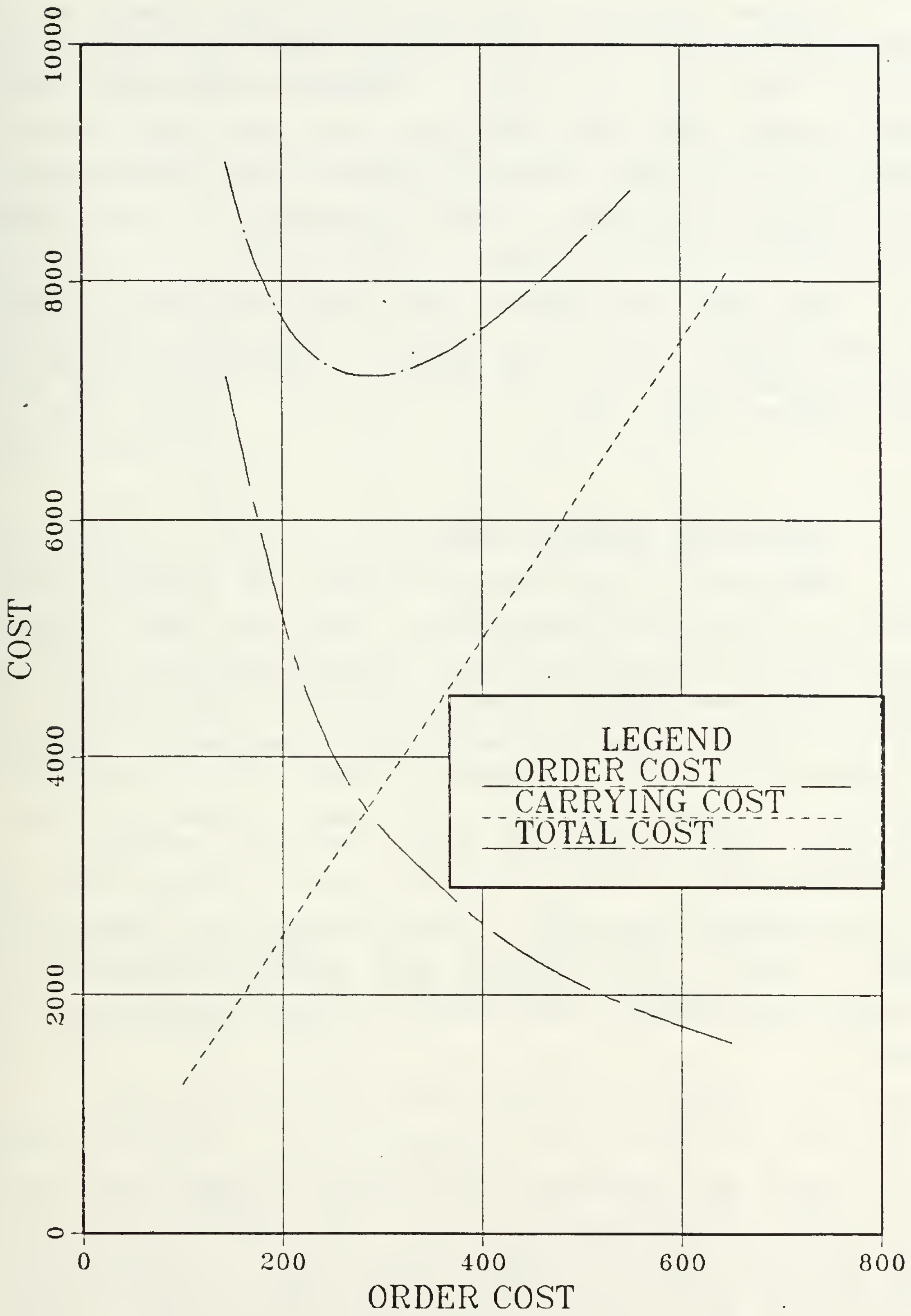

Figure 2.3 Components of the Total Annual Costs

A closer examination reveals that order costs decrease more rapidly than carrying cost increase initially, and thus their sum decreases. This means that there is a positive trade-off since the marginal savings from order costs are more than the marginal increase in carrying costs. But this relationship does not continue. It changes at the point when the marginal savings from order costs are less than the marginal increase in carrying costs.

The optimal order size is that value which minimizes the total annual costs. As can be seen from Table IV and Figure 2.3, the optimal order size is 288 units. This corresponds to an annual order frequency of 18.

2. *Carrying and Backorder Costs*

Safety stock, the stockout cost and the backorder cost depend on the reorder point (ROP). You may recall in the previous chapter it was indicated that, in addition to "how much to order", there was another important facet of the problem to ascertained, this was "when to order". The "when" is provided by the reorder point and is defined in terms of some level of inventory or in number of units of the item. Coyle and Bardi define reorder point as [Ref. 8]:

> "The predetermined inventory level that triggers the need to place an order. This minimum level provides inventory to meet anticipated demand during the time it takes to receive the order".

Under the assumptions of certainty; namely, that the demand rate and lead time are costant, we need to have just enough inventory to last us during the lead time. Therefore, we merely multiply the lead time by the demand rate to get our ROP

value. However, under conditions of uncertainty the inventory world becomes more complicated.

A certain amount of inventory stock is often added to the normal operating stock to hedge against stockouts. It is called safety stock. The difficulty for the inventory manager is to decide how much safety stock is appropriate. Too much safety stock means a high carrying cost, whereas too little safety stock carries with it the penalty of high stockout or backorder costs. The reorder point under conditions of uncertainty needs to allow for safety stock. Therefore, the reorder point becomes the average demand during lead time plus safety stock.

The following example should help provide a better understanding of costs associated with computing safety stock. Let us assume that a company called Pakistan Enterprises has determined that its average demand during lead time is 50 units and that its carrying cost per unit is Rs 5 and its stockout cost is Rs 40 per unit. The company has experienced the probability distribution for demand during the lead time shown in Table V.

Pakistan Enterprises wants to find the value of safety stock that minimizes its total additional inventory carrying costs and stockout costs on an annual basis. The annual carrying cost is simply the unit carrying cost times the safety stock. For example, if the safety stock is 20 units then the additional carrying cost is Rs 5 • 20 = Rs 100.

The stockout cost is more difficult to compute. For any safety stock level, it is the expected cost of being out of stock. This can be computed by first determining the number of times per year the stockout can occur or, in other words, the number of times orders are placed per year. Then the expected number of stockouts which can occur during each order cycle is computed by assuming a value for the ROP and computing the expected number

TABLE V

Probability Distribution of Demand during Lead Time

Number of Units	*Probability*
30	0.1
40	0.2
50	0.4
60	0.2
70	0.1

of units short which occur. For example, when the ROP is 50 (the safety stock is zero) a shortage of 10 units will occur if demand during lead time is 60. A shortage of 20 units will occur if the demand is 70. Thus the expected number of stockouts is (10 units short) •(0.2 probability) + (20 units short) •(0.1 probability) The result is 4 units. If orders are placed six times a year and each stockout costs Rs 40 then the expected stockout cost will be (the expected number of stockout per order cycle) • (unit stockout cost) •(number of orders placed per year) = 4 • Rs 40 • 6 = Rs 960.00.

Table VI summarizes the total costs for the three logical alternative values of safety stock. The safety stock with the lowest total cost is 20 units. With this safety stock, the reorder point becomes 50 + 20 = 70 units. The probability distribution of demand during lead time in table V shows that 70 is the maximum number of units for which demand has occurred. Therefore, for an ROP of 70 or a safety stock of 20 units the system is covered 100% according to the known demand distribution. Obviously, any

additional safety stock would carry an unnecessary carrying cost with it and would not provide any more protection against stockout.

TABLE VI

Total Costs of Safety Stock

Safety Stock	Additional Carrying Costs	Stockout Costs	Total Costs
20	20•Rs5 = Rs100	Rs 0	Rs 100
10	10•Rs5 = Rs 50	10•0.1 •Rs40•6 = Rs240	Rs 290
0	Rs 0	10•0.2•Rs40•6 + 20•0.1•Rs40 •6 = Rs 960	Rs 960

E. SAFETY STOCK WITH UNKNOWN STOCKOUT COSTS

When stockout costs are not available or if they do not apply, then the preceding type of analysis cannot be used. Actually, there are many situations when stockout costs are unknown or extremely difficult to determine. For example, what is the stockout cost of a spare part when it is needed by a military operational unit, be it a ship or an aircraft? There is no easy way of measuring this cost. The cost will be strongly dependent upon the situation under which Navy or any other similar service faces this stockout situation. The stockout cost can be expected to differ greatly for different theaters.

An approach for handling this situation will be presented at the end of the next chapter.

F. NON-DEMAND BASED INVENTORIES

Non-demand based items are those items for which the decision to stock is not based on anticipated demand. Since the Navy may be required to operate in times of conflict without access to production or repair capabilities, and the continued use of a mission essential system may depend upon material that has little or no forecasted requirements, it is prudent to stock certain of these items.

These items fall into two categories in the U. S. Navy, insurance items and numeric stockage objective (NSO) items. An insurance item is a non-demand based, essential item that will not fail in normal usage, but if it does fail or a loss occurs, the lack of a replacement would seriously hamper the operations of a weapon system. NSO stock levels are established for items with a predicted usage too low to qualify as a demand based item, but lack of a replacement item would seriously hamper the operation of a weapon system.

The depth decision for non-demand based items, be these Numeric Stockage Objective items or insurance items, is usually based on the minimum quantity needed for one maintenance action or a quantity of one.

III. INVENTORY CONTROL MODELS

In the previous chapter the author introduced the basic concepts of inventory management and the costs associated with it. Emphasis was placed on the trade-off between the costs, with the intention of providing the reader a basic understanding of inventory control models and how they can serve inventory managers by providing them with an optimal operating doctrine which minimizes the overall cost.

A. SIMPLE EOQ MODEL

In developing a model of any sort, initially the complexity of the real world is ignored and the model is developed under very simplistic assumptions. In an attempt to make a more understandable presentation of the subject we will take the same approach.

To begin our discussion of inventory control, an especially simple model will be studied. This was the model developed in 1915 by Ford Harris and was the first to developed. In this model the rate of demand is assumed to be known with certainty and to be constant over time. In the real world, demands can almost never be predicted with certainty, therefore, demands are usually described in probabilistic terms. This model provides a simple framework for introducing a method of analysis that can also be used in more complicated situation more closely representing the real world.

Following are the assumptions of this simple model:

1) A continuous, constant, and known rate of demand exists.
2) The lead time is constant.

3) No stockouts are permitted.
4) The unit purchase price is the delivered price and is constant.
5) There are no constraints.
6) There is no interaction between items.
7) The above characteristics of the items are constant over the planning period.

The first two assumptions conveniently obviate all apprehensions of stockout.

With these assumptions inventory usage has a saw-tooth shape, as is indicated in figure 3.1. The assumptions for this figure are a weekly usage of 100 units and a lead time of 4 weeks. Suppose that the amount ordered is 1000 units, then all 1000 arrive at one time when an inventory order is received. Therefore, the inventory level jumps from 0 to 1000. In general, an inventory level increases from 0 to Q units when an order of size Q arrives. Because demand is constant over time, inventory drops at a uniform rate over time. This is represented by the sloped lines in figure 3.1. Another order is placed at the ROP of 400 so that just when inventory level reaches zero, the new order arrives and the inventory level jumps to Q units--represented by the vertical lines. This process continues indefinitely over time.

Next we develop a formula for the average annual costs of ordering and carrying. Let

TAC = Total annual costs;
D = Quarterly demand rate; 4D is then the annual demand rate;
Q = The order quantity;
A = The administrative cost of placing an order;
C = The value or cost of one unit of inventory;
I = Inventory carrying cost rate;
IC = Holding cost per unit of inventory per year.

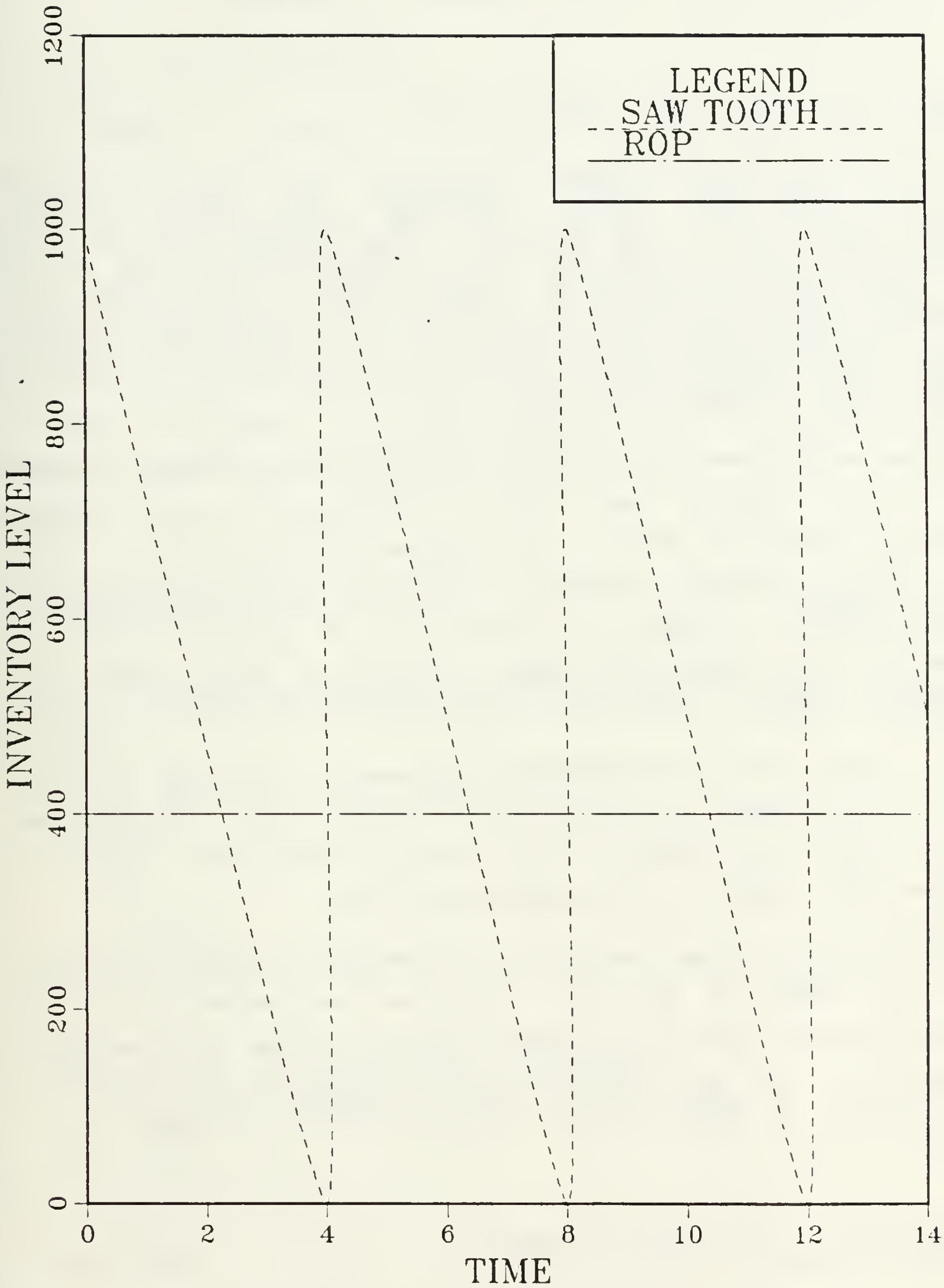

Figure 3.1 On-Hand Inventory.

The Total Annual Cost can be expressed as follows:

$$TAC = IC \cdot Q/2 + A \cdot 4D/Q + 4D \cdot C \qquad \text{(eqn 3.1)}$$

The first term, $IC \cdot Q/2$, represents the annual inventory carrying costs. It is the product of the carrying cost per unit per year (IC) and the average on-hand inventory over time, Q/2. Therefore, the larger the Q the larger would be this first segment of the equation because on the average we will have more inventory on-hand as we increase the order size.

The second term in the above equation represents the ordering costs. The ordering costs are also dependent on the quantity ordered. The ratio 4D/Q is the number of times that an order must be placed during the year in order to avoid shortages and A is the cost of placing one order. As the order quantity is increased, the number of times orders are placed each year decreases. This in turn reduces the total annual ordering costs.

The third term of the equation represents the annual purchase costs. It is simply the unit cost of the inventory item multiplied by the number purchased over the period of one year. This is a constant cost; that is, it is independent of Q.

The economic order quantity is that value of Q which minimizes TAC. We know that TAC is a continuous function of Q and that it has a minimum for some positive value of Q from our example in chapter II. We can determine a formula for optimal Q by differentiating the TAC function with respect to Q as follows:

$$d(TAC)/dQ = 1/2\ IC + A \cdot 4D/Q^2 \qquad \text{(eqn 3.2)}$$

Setting d(TAC)/dQ equal to zero and solving for Q gives

$$Q^* = \sqrt{8\ AD/IC} \qquad \text{(eqn 3.3)}$$

Here we let Q^* denote optimal Q.

In addition to determining how much to order, Q^*, we need to determine when to order. The easiest way to determine when to order is to use the reorder point. As defined in chapter II, it is that level of on-hand inventory such that if we order when our inventory drops to that level then we will receive the order just as we run out of inventory. In our certainty model the length of the lead time is a known constant as is the quarterly demand rate. So all it takes to compute the reorder point here is to multiply the demand per quarter by lead time, L, length in quarters.

Therefore,

$$\text{Reorder Point (ROP)} = D \bullet L \qquad \text{(eqn 3.4)}$$

Let us assume the following to have a better understanding of the equations:

C = Rs 100
I = 25%
IC = .25
A = Rs 200
D = 900 units per quarter
L = 0.1 quarters

If we solve for Q using equation 3.3, we get

$$Q^* = \sqrt{8 \bullet 200 \bullet 900/(100 \bullet 0.25)} = 240 \text{ units.}$$

The reorder point is

$$ROP = 900 \bullet 0.1 = 90 \text{ units.}$$

The Total Annual Cost can be computed, using equation 3.1, as follows:

$$TAC = 25 \bullet 240/2 + 200\ [4 \bullet 900]/240 + 4 \bullet 900 \bullet 100$$

= 3000 + 3000 + 360,000

= Rs 366,000

B. A MODEL FOR UNCERTAINTY OF DEMAND

1. *Causes of Uncertainty*

So far we have assumed the ideal condition of certainty of demand and lead time, therefore we have been able to avoid a stockout. Conditions of certainty, however, seldom exist in real life. There are variety of reasons that contribute to making the real life scenario far from certain. For example, spare parts demand depends in part on when a system fails and needs corrective maintenance. The times when failure occurs are not known in advance.

Variation in lead time is also an important contributing factor. In the case of the Pakistan Navy, where most of the spare parts must be transported over thousands of miles, one of the more important segment of the lead time is the transit time. Other factors that contribute to the variation of lead time are associated with processing of the order and the time associated with its transmittal. If a spare part has to be manufactured or specially fabricated--which is not an unusual occurrence in the Pakistan Navy--then there will obviously be more variation in the time required to obtain an order. Finally there is always the probability of an order of spare parts being lost or damaged in transit.

The variables, mentioned above as having unpredictable or random fluctuations, are termed random or "stochastic" variables.

The only time that a stockout can occur is during the lead time, therefore it is necessary to know the probability of

demand during the lead time. The probability distribution for the demand can either be discrete or it can be a a continuous probability distribution depending upon the nature of the demand. [Ref. 12]

2. *Ordering and Carrying Costs*

When we have random quarterly demand we use the mean of the quarterly demand as D and continue to use the TAC equation 3.1 to decide the annual total expected costs for ordering and carrying the order quantity Q. As a consequence, we can use equation 3.3 to determine the optimal value Q^*.

3. *The Costs of Safety Stock*

The combination of random quarterly demand and lead time creates the need for safety stock to hedge against stock outs between the time an order is placed and received by the Navy Stores depot.

To determine the amount of safety stock, we have to analyze our inventory position and our requirements so as not to keep too much safety stock on hand because that results in excessive inventory holding costs. On the other hand, our safety stock should not be so low as to permit the system to experience excessive stockout costs, resulting in frequent backorders.

As we discussed in Chapter II, we may be able to quantify these costs. If so, then we can write an equation which represents the additional annual costs that result from carrying safety stock and having backorders. It is:

$$TSS = IC \cdot S + K \cdot 4D/Q^* \cdot EBO \qquad \text{(eqn 3.5)}$$

where

TSS = The additional annual costs due to uncertainty

S = Safety stock

K = Unit cost of a backorder

EBO = The expected number of backorders during an order cycle

D/Q^* = The average number of order cycles per year when each order size is Q^*.

4. *Backorder Costs*

In backordering an essential item, expenses may be incurred for special order processing and transportation. The extra order processing time is usually attributable to the additional paper work associated with tracing the movement of backorder, beyond that of the normal processing for regular replenishments. Extra transportation costs are incurred because typically the backorder is specially shipped in a smaller sized shipment with its associated higher rates. Because the Pakistan Navy is heavily dependent on foreign countries for its spares, the transportation costs associated with a backorder is quite high. Each time we run out of a certain spare part that is urgently needed, the part has to be flown to us from other parts of the world at a substantial air freight cost.

This cost can be avoided by having the customer wait until the next order arrives from the suppliers. However, in the case of an essential item this may not be acceptable. Having a Navy customer wait even if the item is not essential is not without its costs. To illustrate its true impact lets look at the business world. There a shortage may also result in lost sales or lost customers. A customer wanting to buy an item that is on backorder may not want to wait. He may take his business to another retailer. The more substituteable a part the easier it is to lose a customer. The direct loss, therefore, is the loss of profit

on the item that was not available when the customer wanted it. In other words, we could calculate the lost profit on an item by multipling the profit per item by the number requested by the customer. For example, if an order was for 100 units and the profit was Rs 10 per unit, the loss would be Rs 1,000.

The above calculation assumes that the customer will return to the store with future business. However, another possibility is that the customer may be lost. That is, the customer permanently switches to another brand or to another store. If a customer is lost a future stream of income is lost. The customer may also tell other customers of his misfortunes and the company may suffer a loss of good will.

Although there isn't the threat of losing future business from the customers in the Naval environment--the obliging factors for avoiding frequent backorders are equally important. Depending upon the importance of the parts in question, frequent backordering may throw the ship's refit out of schedule or may result in delaying or cancelling important operational commitments.

5. *Safety Stock*

Safety stock is defined to be the expected net inventory just before the next order arrives (i.e., at the end of an order cycle) [Ref. 9] Net inventory is defined as the difference between the quantity on hand at any time and the number of backorders at that same time. When the net inventory is positive, there are no backorders; when it is negative, there is no on-hand stock.

The formula for safety stock is

$$S = \sum_{x=0}^{\infty} (ROP - x)\ p(x) \qquad \text{(eqn 3.6)}$$

where

x = Demand during procurement lead time L;

p(x) = The probability of a demand of amount x during L;

ROP - x = The net inventory at the end of L.

This formula can be written as

$$S = ROP - D \cdot L \qquad \text{(eqn 3.7)}$$

where D•L is the mean demand during L, and thus we see that if the value of the product D•L and the value of ROP are known, then the value of S could be quickly determined. Or if S is known, we can compute the value of ROP using:

$$ROP = D \cdot L + S \qquad \text{(eqn 3.8)}$$

6. *Expected Number of Backorders*

The expected number of backorders can be determined by determining those value of x, the demand during lead time, for which the net inventory, ROP - x, is negative. We realize immediately that only if x > ROP do we get a negative net inventory. The expected number of backorders is therefore

$$EBO = \sum_{x=ROP+1}^{\infty} (x - ROP)\, p(x) \qquad \text{(eqn 3.9)}$$

since a negative net inventory corresponds to a positive number of backorders, x - ROP.

7. *Optimization*

The formula for the safety stock and the expected number of backorders can now be included in the equation for TSS. We get

$$TSS = IC(ROP - D \cdot L) + K4D/Q^{*} \cdot \sum_{x=ROP+1}^{\infty} (x - ROP)\, p(x) \qquad \text{(eqn 3.10)}$$

and we see that our variable to be optimized is now ROP.

Because x is a discrete random variable, we cannot take derivatives to find the optimal value of ROP. Instead we must use the calculus of finite differences. We want that value of ROP for which

$$TSS(ROP - 1) > TSS(ROP) \leq TSS(ROP + 1) \qquad \text{(eqn 3.11)}$$

We can express this in an alternative way by considering only the lefthand inequality. We then want the largest value of ROP for which

$$TSS(ROP - 1) > TSS(ROP), \qquad \text{(eqn 3.12)}$$

or

$$\Delta TSS(ROP) = TSS(ROP) - TSS(ROP - 1) < 0 \qquad \text{(eqn 3.13)}$$

The formula for ΔTSS is determined as follows:

$$\begin{aligned} \Delta TSS(ROP) &= TSS(ROP) - TSS(ROP - 1) \qquad \text{(eqn 3.14)} \\ &= IC \cdot (ROP - DL) \\ &+ 4KD/Q^{*} \sum_{x=ROP}^{\infty} (x - ROP)\, p(x) \\ &- IC \cdot (ROP - 1 - DL) \\ &- 4KD/Q \sum_{x=ROP}^{\infty} [x - (ROP - 1)]\, p(x) \\ &= IC - 4KD/Q^{*} \sum_{x=ROP}^{\infty} p(x). \end{aligned}$$

And, since Δ TSS(ROP) < 0 is desired, we have

$$IC - 4KD/Q^{*} \cdot \sum_{X=ROP}^{\infty} p(x) < 0, \qquad \text{(eqn 3.15)}$$

or

$$\sum_{X=ROP}^{\infty} p(x) > ICQ^{*}/4KD \qquad \text{(eqn 3.16)}$$

Now, for computational ease we make use of the identity

$$1 - \sum_{X=0}^{ROP-1} p(x) = \sum_{X=ROP}^{\infty} p(x). \qquad \text{(eqn 3.17)}$$

Substitution into the last inequality above, results in

$$\sum_{X=0}^{ROP-1} p(x) < (4KD- ICQ^{*})/4KD. \qquad \text{(eqn 3.18)}$$

The optimal value of ROP is the largest value of ROP for which this inequality is satisfied. It should be mentioned that if the righthand side is negative then optimal ROP is zero.

8. *The Steps for Computing Optimal Q and ROP*

The following steps comprise the procedure for determining optimal Q and ROP:

Step 1. Use equation 3.3, to compute the value of Q^{*}.

Step 2. Compute the value of V, where

$$V = (4KD- ICQ^{*})/4KD. \qquad \text{(eqn 3.19)}$$

If this is negative then $ROP^{*} = 0$. Otherwise go to step 3.

Step 3. If the probability distribution is discrete then construct a table of ROP values and the associated values of the sum

$$\sum_{x=0}^{R-1} p(x).$$

The potential ROP values correspond to the values of x. From this table, determine the largest ROP for which this sum is less than the value V computed in step 2. This is ROP*.

If the probability distribution is not discrete then use the procedure to be described below in subsection 9 to determine ROP*.

To illustrate these steps the following example should be beneficial:

Assume

D = 900 unit/quarter;

k = Rs 10.00;

IC = Rs 25.00;

A = 200;

and that the first two columns of the following table are the possible demands during lead time and their associated probabilities.

Using equation 3.3 to compute Q*, we get:

$$Q = \sqrt{(8 \cdot 900 \cdot 200)/25}$$
$$= 240 \text{ units.}$$

The second step is to compute V. We get

$$V = (4 \cdot 10 \cdot 900 - 25 \cdot 240)/(4 \cdot 10 \cdot 900)$$
$$= 0.833.$$

TABLE VII

Probability of Demand over Lead Time

	x	p(x)	p(x)
	5100	0.01	0.00
	5200	0.06	0.01
	5300	0.24	0.07
	5400	0.38	0.31
	5500	0.24	0.69
	5600	0.06	0.93
	5700	0.01	0.99
>	5700	0.00	1.00

Going to step 3, we assume ROP = x and compute the sum described in step 3. When we do this we get the third column of Table I. Next we have to compare the value of V, 0.833, to the cumulative values of the table. We find that 0.833 is greater than 0.69, but less than 0.93. Therefore, the optimal ROP value is 5500 units.

9. *Normal Distribution*

If the mean demand during lead time is expected to be large then a normal approximation to p(x) can be used and x can be assumed to be a continuous random variable. If that is so then the inequality 3.20 above changes to the following equality:

$$P(x \leq ROP) = (4KD - ICQ^{*})/4KD. \qquad \text{(eqn 3.20)}$$

and ROP is that value of ROP which satisfies this formula. Again, if the righthand side is negative then $ROP^* = 0$.

To use the normal distribution, we must know the mean and standard deviation of demand during procurement lead time and must find the value of the normal deviate.

The normal deviate z is defined as:

$$z = (ROP - DL)/\sigma \qquad \text{(eqn 3.21)}$$

where σ is the standard deviation of the demand during procurement lead time. From this formula we can write

$$ROP = DL + z\,\sigma \qquad \text{(eqn 3.22)}$$

and we realize that the product $z \bullet \sigma$ is our safety stock. We use this formula to compute ROP^*.

To obtain the value of z we use the following steps:

Step 1. Compute the value of V using the formula of step 2 from subsection 8. If it is negative then $ROP^* = 0$. Otherwise, go to step 2.

Step 2. Use the value of V as the $P(Z \leq z)$ in a standardized normal table to determine z (see section F of Chapter IV for the details for using such a table). Here Z is the random variable which is normally distributed with a mean of 0.0 and a standard deviation of 1.0.

Step 3. Compute ROP^* using equation 3.22.

To illustrate these steps we assume the same example as subsection 8, with the modification that now the mean of the

demand during lead time is normal with a mean of 5400 and a standard deviation, σ, of 107.

Step 1 has already been computed in the previous example and V is 0.833. Next we enter this value into the standardized normal table and determine that z = 0.97.

Step 3. Finally, the third step is to use equation 3.23 to compute ROP^*.

$$
\begin{aligned}
ROP^* &= 900 \cdot 6 + 0.97 \cdot 107 \\
&= 5400 + 101 \\
&= 5501
\end{aligned}
$$

10. *Service Levels*

An alternate approach to determining the reorder point is to base it on a service level. This would be useful if the stockout costs were difficult to determine. In general, a service level is the probability (or the percentage of the time) that one will be able to satisfy demands for a particular item during the procurement lead time. Stated an other way, it is one minus the probability of having a stockout. These relationships, in equation form, are expressed as:

$$\text{Service level} = \text{probability of satisfying a demand} \qquad \text{(eqn 3.23)}$$

which can mathematically be written as

$$
\begin{aligned}
\text{S. L.} &= p(x \le ROP) = \sum_{X=0}^{ROP} p(x) \qquad \text{(eqn 3.24)} \\
&= 1 - \text{probability of a stockout} \\
&= 1 - \sum_{X=ROP+1}^{\infty} p(x)
\end{aligned}
$$

In order to determine the reorder point, it is only necessary to know the probability distribution of demand during the lead time and the desired service level. Table II shows the Service level percentage for the distribution in Table I.

Even the inventory manager does not know the value of the stockout costs, he usually has a sense about what service level he would like to provide. If, in the example, he decides he wishes to have a service level of at least 90% then he would select on ROP of 5500. If he wants at least a service level of 95% then he would select an ROP of 5600.

TABLE VIII

Service Level Percentage

ROP	$p(x)$	$\sum_{x=0}^{ROP} p(x)$	*Service Level%*
5100	0.01	0.01	1
5200	0.06	0.07	7
5300	0.24	0.31	31
5400	0.38	0.69	69
5500	0.24	0.93	93
5600	0.06	0.99	99
5700	0.01	1.00	100

IV. FORECASTING AND DEMAND DISTRIBUTION

A. INTRODUCTION

Chapter IV's presentation of inventory model emphasized that the quarterly demand and lead time length were random variables. As a result, their values are never known for certain until they are recorded. However, it is possible to forecast the mean and standard deviation of their values. This chapter will describe models which can be used in this forecasting. It will also describe how the forecasts should be used to estimate the mean and standard deviation of demand during procurement lead time. Finally, two probability distributions to describe this demand are proposed.

B. FORECASTING QUARTERLY DEMAND

Two forecasting models are worthy of consideration. They are the moving average and the exponentially weighted average.

1. *Moving Average*

The simplest model for estimating the mean value of a demand is the four-quarter moving average. This is calculated simply by dividing the sum of the demands in the last four quarters by the number of quarters in question. Equation 4.1 is the formula for this model.

$$F(n+1) = [D(n) + D(n-1) + D(n-2) + D(n-3)]/4 \qquad \text{(eqn 4.1)}$$

Here $F(n+1)$ = Forecasted demand for the next quarter;
$D(n)$ = Actual demand of the present quarter;

D(n-1) = Actual demand during the current minus 1 quarter;

D(n-2) = Actual demand during the current minus 2 quarters;

D(n-3) = Actual demand during the current minus 3 quarters.

When an item is initially placed into the inventory system no historical data is available to use in the formula. Even at the end of the first three quarters data is still missing. This shortcoming can, however, be easily overcome by using an "initialized moving average". An initialized moving average is calculated by dividing the sum of the data available by the number of quarters from which that data is drawn until four quarters of data have been accumulated; then from the fourth period onwards equation 4.1 can be used. For instance, for the first time period, the forecast of the mean value for the second time period would be the same as that value of the demand which occurred in the first period. The forecast for the third period would be the sum of the demand values in the first two quarters divided by two. The forecast for the fourth period would be the sum of the first three quarters demand values divided by three.

Moving averages based on more than four quarters may be appropriate. However, recent experience with practical applications has suggested that four quarters is a good interval to use. This forecast is easy computationally, but it does have its limitations. To get a moving average it is necessary to store demand data for the four quarters being used in the equation.

2. *Exponentially Weighted Average*

The disadvantage of storing four quarter of demand information for the moving average can be overcome by using a model known as the "exponentially weighted average" or exponential smoothing. In the sense in which it is used here, weighting

means the proportion of the eventual value of the average being formed that each individual piece of data contributes. Referring again to the four quarter moving average, it is evident that each piece of data contributes one fourth of its value to the value of the moving average. As there are four pieces of data it is also evident that the sum of the weights is one. This notion is also used in the exponentially weighted average.

In the exponentially weighted average, instead of giving equal weight to each of the four quarters and ignoring all the remaining information by giving these zero weights as is done in the moving average, all past observations are given some weight. The weightings gradually become smaller as the data becomes older.

The exponentially weighted average model is described by the following formula:

$$F(n + 1) = \alpha \quad D(n) + (1 - \alpha) \; F(n), \qquad \text{(eqn 4.2)}$$

where $0 < \alpha < 1$ and

$F(n)$ = The exponentially weighted average calculated for the current period.

$D(n)$ = The value of the quarterly demand which occurred during the current period.

Having defined the exponentially weighted average it is now possible to examine its advantages over the moving average. These are as follows:

a) Only one piece of information need be retained between forecasts; that is, $F(n + 1)$ which becomes $F(n)$ when the next quarter's forecast is made. For the moving average, $(m - 1)$ pieces of information must be retained, if m is the number of quarters on which the moving average is based.

b) The sensitivity of the exponentially weighted average can be altered at any time by changing the value of α. An increased value of α gives more weight to recent data and, therefore, makes the average more sensitive. Conversely, a smaller value of α gives less weight to recent data and makes the average less sensitive to the recent data. For the moving average, the sensitivity can only be altered by changing m, the number of quarters on which the moving average is based, and this cannot generally be effected over a single time period as with the exponentially weighted average.

Robert G. Brown in his book "Statistical Forecasting for Inventory Control", [Ref. 5] indicates that the value of α used in forming an exponentially weighted average should not usually exceed 0.3 and, if it appears that a higher value of α is required, the assumption that D(n) is drawn from a stationary distribution is likely to be invalid. The U.S. Navy uses an α value of 0.2 unless a change in the mean of the distribution of D(n) is suspected. In that case an α value of 0.4 is used.

As with the method of moving averages, when beginning a forecasting system with the exponentially weighted average method, an initial estimate using the same moving average approach of one, two, three, and the four periods has been found to be practical by the U.S. Navy. The first full four-quarter average is then the first F(n) value of the exponential weighted model.

At present the four quarter moving average is considered to be more desireable than the exponential unless data storage is a problem.

C. FORECASTING VARIATION IN QUARTERLY DEMAND

The mean absolute deviation (MADD) is used to estimate the standard deviation of quarterly demand. The formula for the sample standard deviation is the square root of the sum of the squared differences of the observed demands from their forecasted mean, divided by the number of observation less one. Such a computation is time consuming and requires many observations. A much simpler way of estimating this deviation is to use the fact that it is approximately equal to 1.25 times MADD. It can be shown that this result is exact in the case of quarterly demand being normally distributed.

The mean absolute deviation is defined as the average of the absolute forecasting errors, absolute indicating that one regards the error always as positive irrespective of its actual polarity. The mean absolute deviation for the four-quarter moving average forecast is obtained by the following formula:

$$\begin{aligned} MADD(n+1) = 1/4\ [&|D(n) - F(n)| \\ &+ |D(n-1) - F(n-1)| \\ &+ |D(n-2) - F(n-2)| + |D(n-3) - F(n-3)|]. \end{aligned} \qquad \text{(eqn 4.3)}$$

The MADD forecast formula for the exponential moving average is:

$$\begin{aligned} MADD(n+1) = \ &\alpha|Dn - F(n)| \\ &+ (1 - \alpha)MADD(n). \end{aligned} \qquad \text{(eqn 4.4)}$$

D. CHANGING THE EXPONENTIAL WEIGHT

If a steady trend in demand is suspected and the exponential moving average model is being used, a test for this trend is needed. The U.S. Navy uses the following formula for the trend test parameter T.

$$T = 2\,[D(n) + D(n-1)] \,/\, [D(n) + D(n-1) + D(n-2) + D(n-3)]. \qquad \text{(eqn 4.5)}$$

Then if

$T < 0.9$ and $D(n) \le F(n)$, change α from 0.2 to 0.4;

$T > 1.1$ and $D(n) \ge F(n)$, change α from 0.2 to 0.4;

otherwise use $\alpha = 0.2$.

E. FORECASTING LEAD TIME

The decision of when to order more inventory depends on the lead time involved for the item in question. The lead time starts when the inventory manager detects that the inventory has dropped below the reorder point and ends with the receipt of the ordered material into inventory.

The lead time typically consists of two parts, the time it takes the inventory manager and procurement people to prepare the order and negotiate with the manufacturer and the time it takes the manufacturer to produce and deliver the order. The first part is usually referred to as administrative lead time (ALT) and the latter is referred to as production lead time (PLT). The sum is called the procurement lead time (L).

The U.S. Navy has found that forecasting procurement and production lead times are the easiest to do because the times when a reorder point is passed, when an order is received, and the manufacturer's estimate of his production time are easy to obtain.

As above, there are two parts to be forecasted. One is the L and the other is the mean absolute deviation for L(MADL).

If we denote the compute forecast of L as L(n) then we can use the following formula to determine its value, assuming the exponential weighting model.

$$L(n + 1) = \alpha \quad L(\text{observed}) \qquad \text{(eqn 4.6)}$$

$$+ (1 - \alpha) L(n)$$

where

$$L\ (\text{observed}) = \frac{(\text{sum of days for each buy in this quarter})}{(\text{number of buys in this quarter})\cdot 91} \qquad \text{(eqn 4.7)}$$

We multiply the denominator by 91 because there are 91 days in a quarter.

To illustrate the use of equation 4.7 let us assume that we received two buys in this quarter. One buy took 265 days from the time of initiation to the receipt of goods, and the other took 310 days. Using equation 4.7, to compute L (observed), we get

$$L\ (\text{observed}) = (265 + 310)/(2\cdot 91)$$
$$= 3.16 \text{ quarters.}$$

The U.S. Navy bases the α value on their perceived validity of L(n).

α = 0.2 if the last buy arriving before this quarter was one or two quarters ago.

α = 0.5 if the last buy arriving before this quarter was three or four quarters ago.

α = 1.0 if the last buy arriving before this quarter was beyond one year ago.

While these α values are not necessarily applicable to the Pakistan Navy, these do provide some idea of what the rough range of α values should be.

The following equation is used for computing the forecast of the MADL:

$$MADL(n + 1) = \alpha\cdot|L(\text{observed}) - L(n)| \qquad \text{(eqn 4.8)}$$

$$+ (1 - \alpha)\ \mathrm{MADL}(n).$$

F. PROBABILITY DISTRIBUTIONS

The U.S. Navy uses the normal distribution for the fast moving or high rate demand items and the Poisson distribution for the slow moving items. This is done irrespective of the value of the items. Experience shows that of the entire spares requirement 75% to 80% are slow moving and the remaining 15% to 20% are the fast moving items. Typically the fast movers account for 80% of the demands.

In the ensuing discussion the following notation will be used:

L = Lead time forecast (considered the current average value for lead time), in quarters;

D = Quarterly demand forecast (considered the current average value for the quarterly demand), in units;

σL = Standard deviation of lead time (computed from the MAD forecast for lead time), in quarters;

σD = Standard deviation of quarterly demand (computed from the MAD forecast for quarterly demand), in units.

1. *The Normal Distribution*

Among the many continuous probability distributions used in statistics, the normal distribution is the most important. In studies dating back to the 18th century, it was observed that discrepancies between repeated measurements of the same physical quantity displayed a surprising degree of regularity; their pattern

(distribution) was found to be closely approximated by a certain kind of continuous distribution, referred to as the "normal curve of errors" and attributed to the laws of chance. The mathematical properties of this continuous distribution and its theoretical basis were first investigated by Pierre Laplace (1749 - 1827), Abraham deMoivre (1667 - 1745), and Karl Gauss (1777 - 1855). [Ref. 8]

An important feature of the normal distribution is that it can be completely characterized by its mean and standard deviation. In fact, the equation of the normal distribution, known as the density function is represented by the following formula:

$$f(x) = (\sigma\sqrt{2\pi})^{-1} \, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{(eqn 4.9)}$$

for all real values of x. Here μ represents the mean and σ represents the standard deviation.

The graph of the normal distribution is a bell-shaped curve that extends to infinity in both directions. Although this may not be apparent from a small drawing that one usually sees in text books, the curve comes closer and closer to the horizontal axis without ever touching it, no matter how far we might go in either direction away from the mean. However, it is seldom necessary to extend the tails of a normal distribution very far because the area under the tails lying more than 4 or 5 standard deviations away from the mean is, for most practical purposes, negligible.

In practice, we obtain areas under normal curves by means of a standard normal table. This table gives the areas under various portions of the normal curve for that normal distribution having a mean of zero and a standard deviation of one. Such a table is presented in Appendix A of [Ref. 12] To convert this information to that associated with a random variable x, normally distributed with a general mean μ and standard deviation σ, we use the following formula:

$$z = (x - \mu)/\sigma \qquad \text{(eqn 4.10)}$$

where z is called the normal deviate. The units along the z scale are referred to as the standard units.

To find areas under normal curves whose mean and standard deviation are not 0 and 1, we have only to convert any x value of interest into z and then use the standard normal distribution table. The entries in this table are the areas under the standard normal distribution between $-\alpha$ and non-negative values of z. To determine the probability that z lies between two positive values we take the difference between the area values for the larger and smaller values of z. Although the table has no negative values of z, these are not difficult to compute by the virtue of the symmetry of a normal curve about its mean. We can find the area under the standard normal distribution, say, between $z = -1.81$ and $z = 0$, by looking up instead the areas for $z = 0$ and $z = 1.81$. The answer is then the difference in the areas; namely, 0.9649 - 0.5000, or 0.4649.

Applying this principle in the our field of inventory management we have the mean during procurement lead time is represented by

$$\mu = D \bullet L. \qquad \text{(eqn 4.11)}$$

The variances of the demand and the lead time-- which are the standard deviations squared--are represented

$$\sigma D^2 = 1.57\ (MADD)^2, \qquad \text{(eqn 4.12)}$$

$$\sigma L^2 = 1.57(MADL)^2. \qquad \text{(eqn 4.13)}$$

The variation of demand during lead time can then be represented by [Ref. 9]

$$\sigma^2 = L \bullet \sigma D^2 + D^2 \bullet \sigma L^2 \qquad \text{(eqn 4.14)}$$

and the standard deviation is then the square root of equation 4.14.

If we are interested in the probaility that demand during lead time is less then or equal to some value, say x, we first compute z using this x value and μ and σ as computed from equations 4.11 and 4.14 in the equation 4.10. Then we look up the area associated with z in the table of the standard normal. This area is the desired probability value.

2. *Poisson Distribution*

The Poisson distribution is a discrete distribution represented by the following formula:

$$p(x) = (DL)^{x} e^{-DL} / x! \qquad \text{(eqn 4.15)}$$

where p(x) is the probability for the value of x. Here e is the irrational number which is approximately equal to 2.71828. In the Poisson distribution, the random variable x can take on the infinite set of integer values of x = 0, 1, 2, 3, 4,... Practically speaking, this will not pose any problems, since the probabilities usually become negligible (very close to zero) after the first few values of x.

Finally, we note that we do not need a standard deviation when we use the Poisson distribution. Only the mean of the

demand during lead time, DL, is needed. A recursive form basedon equation 4.15 can be used when computing p(x) with a programmable calculator or a computer. The following steps would be used.

First we compute

$$p(0) = e^{-DL} \qquad \text{(eqn 4.16)}$$

Then, to get p(1), we observe that

$$p(1) = DL \cdot e^{-DL} = DL \cdot p(0). \qquad \text{(eqn 4.17)}$$

Next,

$$p(2) = (DL)^2 \cdot e^{-DL}/2! = (DL/2) \cdot p(1), \qquad \text{(eqn 4.18)}$$

$$p(3) = (DL)^3 \cdot e^{-DL}/3! = (DL/3) \cdot p(2). \qquad \text{(eqn 4.19)}$$

The general form is therefore

$$p(x) = (DL/x) \cdot p(x - 1) \qquad \text{(eqn 4.20)}$$

The following example illustrates these steps.

Assume

D = 0.1 units per quarter;
L = 6 quarters;
Then
DL = 0.6 units.

Therefore,

$$p(0) = e^{-0.6} = 0.5488$$

$$p(1) = 0.6 \cdot (0.5488) = 0.3293$$

$$p(2) = (0.6/2) \cdot (0.3293) = 0.0988$$

$$p(3) = (0.6/3) \cdot (0.0988) = 0.0198$$

$$p(4) = (0.6/4) \cdot (0.0198) = 0.0029$$

$$p(5) = (0.6/5) \cdot (0.0029) = 0.0004$$

$$p(6) = (0.6/6) \cdot (0.0004) = 0.0 \text{ (approximately)}$$

and $p(x)$ for $x > 6$ is approximately zero.

V. PROVISIONING

A. INTRODUCTION

With the rapidly changing technology, today's Navies need newer hardware and weapons systems much more frequently. The support of these new systems has to be planned well in advance if the new equipment is to be effectively used. Supply support is only one of the logistic support elements that must be planned to ensure that a new weapon is adequately supported. Initial supply support is achieved through a provisioning process which must begin very early in the acquisition phases of the new hardware.

This chapter introduces the analytical concepts which would help in providing initial supply support to a complex new weapon system and equipment. The chapter begins with a demand forecasting model. It then considers a model used by the U.S. Navy to decide on initial range and buy quantities of the items to be stocked.

B. PROVISIONING INTERVAL

The following four key dates should be considered in any provisioning problem:

1. POC: The date of planned operational capability (the provisioning package should be in place by this date).
2. POC + TR: The time at which the first replenishment buy is initiated.
3. POC + TR + L: The time at which the first replenishment buy is received.

The primary concern of provisioning is to determine the number of units of each component of a system to be purchased at the time POC - L (and possibly purchased at additional times between POC - L and POC). In order to reduce the likelihood of stockouts, the provisioning buy made at time POC - L should be sufficiently large to satisfy anticipated demands in the interval from POC to POC + TR + L. If good forecasts of demand during that interval and of the times POC, TR and L were available, the solution of the provisioning problem would be fairly routine. However, the provisioning problem is characterized by a great deal of uncertainty with respect to the failure rates of the new equipment, the procurement lead time L, and the time TR. During the time interval from POC to POC + TR + L, the population of the equipment will grow as additional units are installed in the service. This will cause the aggregate failure rates to increase over the time interval of concern in the provisioning problem. Actual installation schedules may be subject to great deal of uncertainty. The failure rate estimates often represent only theoretical guesses by engineers of the actual the failure rates. The time lags are very large (the time interval from POC - L to POC + TR + PLT may be of the order of four years or more). Thus it is not surprising that the provisioned quantities frequently do not match the demands very well.

C. TIME WEIGHTED AVERAGE MONTH'S PROGRAMME

The schedule of installation of the weapon system will be referred to as the Programme in this section. Programme data is used in the computation of the levels of support stock. Those levels should be developed based on a time weighted average programme (TWAMP) spanning the Programme Time Base (PTB). The value of PTB depends on the environment. In the Pakistan Navy it should probably be equal to the procurement lead time (L).

TWAMP is computed by determining the area under the curve of the total installated population over time from POC to POC + PTB and then dividing it by the length of the PTB. This results in an average number of end items to be supported over the PTB. This average is denoted as the initial TWAMP or TWAMPi.

The following formulae are provided for computing this initial TWAMP. They obtain the area by summing the vertical slices of the area spanning each month. Any installations occurring in a specific month are assumed to begin operation in the middle of the month.

$$A_t = \begin{cases} \dfrac{I_1}{2} & \text{for } t = 1 \\ \displaystyle\sum_{K=1}^{t-1} I_K + \dfrac{I_t}{2} & \text{for } t \geq 2 \end{cases} \qquad \text{(eqn 5.1)}$$

here

t = The number of months after POC;

A = The area of the time slice of the curve described above for month t;

Ik = The number of units of the end item to be installed in in month k.

Then

$$\text{TWAMPi} = \frac{\displaystyle\sum_{t=1}^{PTB} A_t}{PTB}. \qquad \text{(eqn 5.2)}$$

The initial annual demand rate for a given spare part can be computed by using the following formula:

$$Di = \text{TWAMPi} \cdot N \cdot \text{BRF}, \qquad \text{(eqn 5.3)}$$

where

N = the number of units of a given replaceable part in the end item;

BRF = the best replacement factor which is, in fact, the estimated failure rate of one unit over a year.

The BRF is initially based on a technical replacement factor (TRF) which is the contractor's estimate of the attrition rate.

The COSDIF formula to be described in the next section requires an initial estimate of the steady state annual demand rate (Dss). This is computed by first determining the total number of end items which are expected to be installed by the end of the procurement lead time.

$$Dss = TWAMPss \bullet N \bullet BRF \qquad \text{(eqn 5.4)}$$

The development of the provisioning budget, to be discussed in a later section, requires that an estimate of the initial demand during the procurement lead time (L) plus one quarter be computed. This is then the buy quantity value to be used to develop the budget. For consumable items the formula is:

$$D(L+1) = Di \bullet (L+1)/4 \qquad \text{(eqn 5.5)}$$

D. THE COST DIFFERENCE FORMULA (COSDIF)

The COSDIF Formula was first introduced by Alan Kaplan of the United States Army's Inventory Research Office. He was a member of a team that developed a Department of Defence Instruction on the subject of initial provisioning. This formula is derived from the U S Army's wholesale range model. It can be

viewed as the results of a simple decision model as illustrated in figure 6.1. Here the demand development period (DDP) is assumed to be two years. After that time the observed demand is expected to be sufficient to provide good forecasts.

	States of Nature	
Decision	*No demand during DDP*	*Demand during DDP*
Make a provis-ing buy	Cost of procurement plus two years' holding costs	Two years of average annual variable costs
Do not make a provision ing buy	No costs	Costs of spot buys during first year and average annual variable costs for second year

Figure 5.1 A Provisioning Decision Matrix

The expected costs of each decision can be evaluated when all the costs and the probability values for the states of nature are known. That decision which corresponds to the least expected costs is then the optimal one. The difference between the expected costs associated with each decision can be conveniently used to make the decision of whether or not to stock an item. This difference will be referred to in the rest of this section as the COSDIF. The following equation expresses analytically the differences between the expected costs of stocking and not stocking.

$$\text{COSDIF} = (\text{Do}|\text{Dss}) \cdot [A + 2IC(\text{ROP} + Q)] \quad \text{(eqn 5.6)}$$
$$+ (1-\text{Do}|\text{Dss})\cdot[4AD/Q + ICQ/2 + \text{Dss}\cdot\text{CI}]$$
$$- (1-\text{Do}|\text{Dss})\cdot[\text{Dss} \cdot(\text{CSP} + K\cdot\text{PLT}/4) + \text{Dss}\cdot C\cdot P]$$

here:

Dss	=	Steady state annual demand;
Do|Dss	=	probability of no demand in two years, given an annual steady state demand forecast (total) of Dss;
A	=	Cost of procurement;
I	=	Carrying cost rate;
C	=	Unit price;
ROP	=	reorder point quantity;
Q	=	Optimal order size;
CI	=	Cost of issuing stock;
CSP	=	Cost of a spot procurement;
PLT	=	Production lead timein quarters;
K	=	Shortage cost per unit per year;
P	=	Spot buy premium rate.

If the COSDIF value is negative, then the costs associated with not making a buy are greater than those for making the buy. It is, therefore, optimal to make the buy. If the COSDIF is positive, then making a buy is more costly. If the COSDIF is zero, then either decision would be optimal but not making a buy is less work for the provisioner so no buy is made.

The COSDIF formula serves as the range model in the development of the provisioning budget. Therefore, it is of immense importance to the managers in determining their initial stockage priorities.

E. DETERMINING THE BUY QUANTITY AND BUDGET

If an item has been determined by the COSDIF as appropriate to stock, the buy quantity needs to be determined. The U.S. Department of Defence Instruction specifies that this quantity be equal to the demand expected over a period of time consisting of the forecasted replenishment procurement lead time plus one quarter. The extra quarter of demand is viewed as safety stock. Equation 5.5 is used to compute this.

Those items which fail to meet the COSDIF criterion are next re-examined to determine if they can be identified as insurance or Numeric Stockage Objective (NSO) items. If, so the buy quantity for each item is taken to be one minimum replaceable unit (MRU).

The cost of buying each item is the product of C and D(L + 1) or just C depending on whether the item is demand based or is an insurance or NSO item.The total value of a provisioning package is then obtained by summing these procurement costs over all of the items. This sum is then the proposed provisioning budget.

In the U.S. this sum is also established as "the budget constraint". The monetary value so determined is considered as a firm upper limit on the amount of money that can be spent to purchase those items that are to be stocked. The actual range and buy quantities of items that are stocked may deviate from the values used to determine the budget but approval of the procedures used is required.

F. A NUMERICAL EXAMPLE

This section presents an example which illustrates the steps for developing a provisioning budget.

Suppose that a installation schedule for a new weapon system known as Mk 98 is as follows:

Months:	*O*	*N*	*D*	*J*	*F*	*M*	*A*	*M*	*J*	*J*	*A*	*S*
FY-86:			1	1	2	2	2	2	3	4	4	4
FY-87:	6	7	7	7	8	8	9					

PTB = L = 5.4 quarters.

Start by determining the values of TWAMPi and TWAMPss. The following sequence of the numbers are the A values for the respective months.

FY-86--- 0.5, 1.5, 3, 5, 7, 9, 11.5, 15, 19, 23

FY-87--- 28, 34.5, 41.5, 48.5, 56, 64, 72.5, 77,
77, 77, 77, 77

Therefore, TWAMPss = 77 since it represents the total installation to be made.

TWAMPi is calculated by first summing the A values over the months of the PTB and then dividing the sum by the PTB value.

$$\sum_{t=0}^{PTB} A = 0.5 + 1.5 + 3 + 5 + 7 + 9 + 11.5 + 15 + 19 + 23 + 28 + 34.5 + 41.5 + 48.5 + 56 + 64$$

TWAMPi = 367/PTB = 367/16 = 22.94

Having calculated both types of TWAMP, let us now assume there are four demand based consumable repair parts for the Mark 98 which are being provisioned. Suppose that the data for each and the sign of the COSDIF value are as follows:

When determining the provisioning budget for consumeables one should remember that only items having a negative COSDIF can be used for budget generation.

The following steps are involved in determining provisioning budget:

1. Decide whether item is insurance or demand based item.

2. If it is an Insurance based item then buy one MRU.

TABLE IX
Data of Repair Parts

Item No	COSDIF	N	BRF	Cost	MRU
1	(-)	3	0.35	Rs 150	1
2	(-)	10	0.15	10	2
3	(-)	1	0.15	30	1
4	(+)	2	0.10	5	1

3. The cost of each item is computed by the following equation:

$$\text{Cost} = C \bullet MRU \qquad \text{(eqn 5.7)}$$

or

$$\text{Cost} = C \bullet D(L + 1) \qquad \text{(eqn 5.8)}$$

TABLE X

Cost for Consumables with Negative COSDIF

Item	D (L + 1)	C•D(L + 1)
1	22.94•3•0.35•1.6 = 29	Rs 5850
2	22.94•10•0.15•1.6 = 55	550
3	22.94•1•0.15•1.6 = 5	150
Total Consumeable Budget		Rs 6550

VI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

The basic concepts of inventory management for consumable items were introduced in Chapter II. It began by defining the purpose for carrying inventories, and then talked about both demand based inventories and non-demand based inventories. This chapter discussed the "operating doctrine" or the all important question of "how much to order" and "when to order" for demand based items. The objective of the optimal operating doctrine was minimizing total annual costs. In subsequent subsections the components of the total annual costs; the administrative order costs, and carrying costs were defined and discussed. Assuming variables like demand quantity and the lead time are constant. The optimal operating doctrine is determined from trade-offs between the various components of the total annual cost. This means striking a balance between the two cost components to find the lowest total cost for the system. This point was highlighted with the help of an example.

The concept of inventory management under conditions of uncertainty was then introduced. This made the situation a little more complicated and necessitated the introduction of safety stock and the related cost components, carrying costs for safety stock and stockout costs. In an uncertain environment, these two components are very real and need to be balanced. This point was illustrated by an example.

A discussion of non-demand based inventories completed this chapter. The U.S. Navy categorization of these non-demand based inventories into insurance items and Numeric Stockage Objective (NSO) items was discussed. The depth for these types

of items is usually based on the minimum quantity needed for one maintenance action or a quantity of one.

Chapter III formalized the concepts of chapter II into mathematical inventory control models. Two models were developed. The first, the simple economic order quantity model (EOQ), assumed a constant and known rate of demand and a constant lead time not permitting stockouts. The formulae for total annual cost, the optimal order quantity Q, and the reorder point (ROP) were derived and their use was illustrated by an example.

The second model considered uncertainty of demand. The causes of the uncertainty were discussed and how this uncertainty required the building up of a safety stock. The EOQ model was modified and used to determine the optimal value of Q. The additional annual costs associated with safety stock were formulated from detailed discussion of backorder costs and safety stock. The optimal reorder point was then derived from these additional annual costs. These resulting model formulae were then used in an example to illustrate the steps for determining the optimal order size and the optimal reorder point under uncertain conditions.

Chapter III ended with a discussion of an alternate approach for determining the reorder point which was based on the concept of service level. The service level concept was discussed and the procedure and formulae for computing the reorder point were presented. Finally, an example was provided to illustrate the application of the service level concept.

Chapter IV dealt with forecasting lead time demand and probability distribution for this demand. Lead time demand consists of two components, the quarterly demand rate and the procurement lead time. Two forecasting models, the Moving Average and the Exponentially Weighted Average are presented. The moving average is a simple model for estimating the mean value of quarterly demand based on four quarters of observed demands.

The exponentially weighted average is a model that overcomes the disadvantage of storing four quarters of demand information. It forecasts based on one quarter's demands and its forecast. The weight can be changed if trends are detected. This model was also suggested for forecasting the procurement lead time. Another model, based on the mean absolute deviation (MAD) of forecasting errors is suggested for estimating the standard deviations of quarterly demand and procurement lead time.

The second section of Chapter IV proposed two probability distributions for demand during lead time and discussed which distribution should be used for what volume of demand. The distribution are the normal and the Poisson. Their formulae were presented and the details for their use were described. The chapter concluded with an example of computing the probabilities for the Poisson distribution using a recursive formula.

Chapter V discussed provisioning of depot stock to provide initial backup supply support to a newly inducted equipment or weapon system. This chapter began with the introduction of key dates that are considered important for provisioning. This was followed by a discussion of demand of how to determine initial demand forecasts. A model called the Cost Difference Formula was then described determining the range of items to buy. Finally, a model for computing the buy quantity for those items to be bought was presented and steps for computing the procurement budget were shown. An example illustrated the steps was also presented.

B. CONCLUSIONS

Total inventory costs can be reduced through the implementation of various models for cost optimization. We can establish a conscious policy to increase inventory as long as the additional expenditure for inventory leads to compensating cost

reductions in other areas. For example, a high inventory level significantly decreases the risk of backorder and stockout costs.

The cost of storage vary depending upon the type of inventory. Raw material invariably requires minimal storage facilities compared to finished goods which need rather sophisticated facilities and may even require temperature and humidity control. In the case of the Naval Stores Depot most of the inventory consists of finished items requiring careful handling and, at times, regulated temperature.

Ascertaining the stockout cost is an important facet of inventory control but is very complicated. To evaluate this cost, inventory managers need to make some important decisions. They should decide what level of protection is required for the equipment in question over the procurement lead time. This level will then imply the value of shortage costs to be used in a cost minimization model.

The fixed quantity model for cost minimization has been discussed at length in this thesis. Customarily this type of model is used when dealing with the more important inventory items since these items require a close scrutiny for ascertaining their inventory levels and the reordering point. Close surveillance is expensive and should only be done for the more important items. An alternative model would be to review inventories at fixed intervals and order variable quantities. That model does not require the close monitoring of inventory levels. Such a model is often used for less expensive items. This thesis did not discuss the fixed interval model for fear of introducing too many diverse concepts all at once and confusing the readers.

C. RECOMMENDATIONS

Inventory management is a complicated and a perilous task, but the implementation of scientific inventory models should make this task easier and the managers more proficient. A systematic approach towards implementing the various models of this thesis should relieve the Pakistan Navy of many of its inventory control problems.

It is important to introduce the inventory managers to the concepts of scientific inventory control models. These managers should be made conscious of the strides that the inventory world has taken. Once conscious of the importance of holding a systematized inventory control system their interest should be channelled towards trying to implement the models described in this thesis.

A first step could be to use the model presented in Chapter IV for forecasting quarterly demands and lead times. This is a good starting point because the demand history does exist at the Naval Stores Depot. A second step would be to attempt to quantify the various costs needed by the model. Once these are quantified the steps for computing the optimal order quantities and reorder points could be programmed.

After the consumable model is implemented and experience is gained in its use, it would be appropriate to develop a repairable model. It would also be appropriate to develop a fixed interval model for both slow moving consumable and repairable items.

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Thesis
H3584 Hayat
c.1 An inventory model
for the Pakistan Naval
Store Depot.